From Preferential Input

To Proportional Output

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Presentation Plan

- ❑ Influence of Preferential Ballot over Voting Behavior

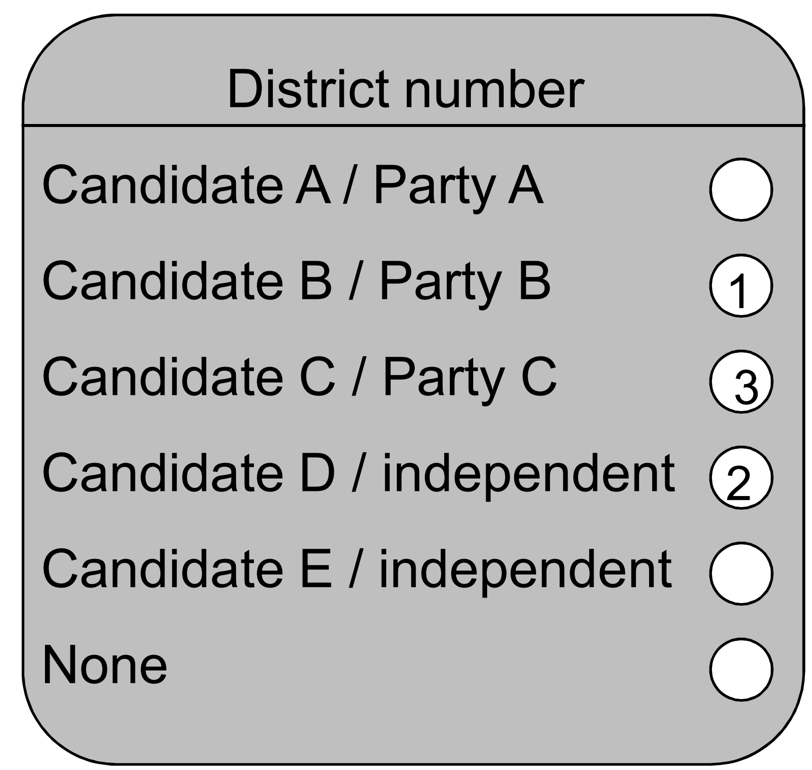
- ❑ Influence of Proportional Representation over Voter Participation

- ❑ Proposed Solution (SPPA)
  - Preferential ballot
  - Rallying procedure
  - Integral proportional representation
  - Reproducible tie-breakers

- ❑ Conclusion
  - Big Data and Artificial Intelligence
  - Impact of rich feedback on voter behavior

Enhanced Preferential Ballot

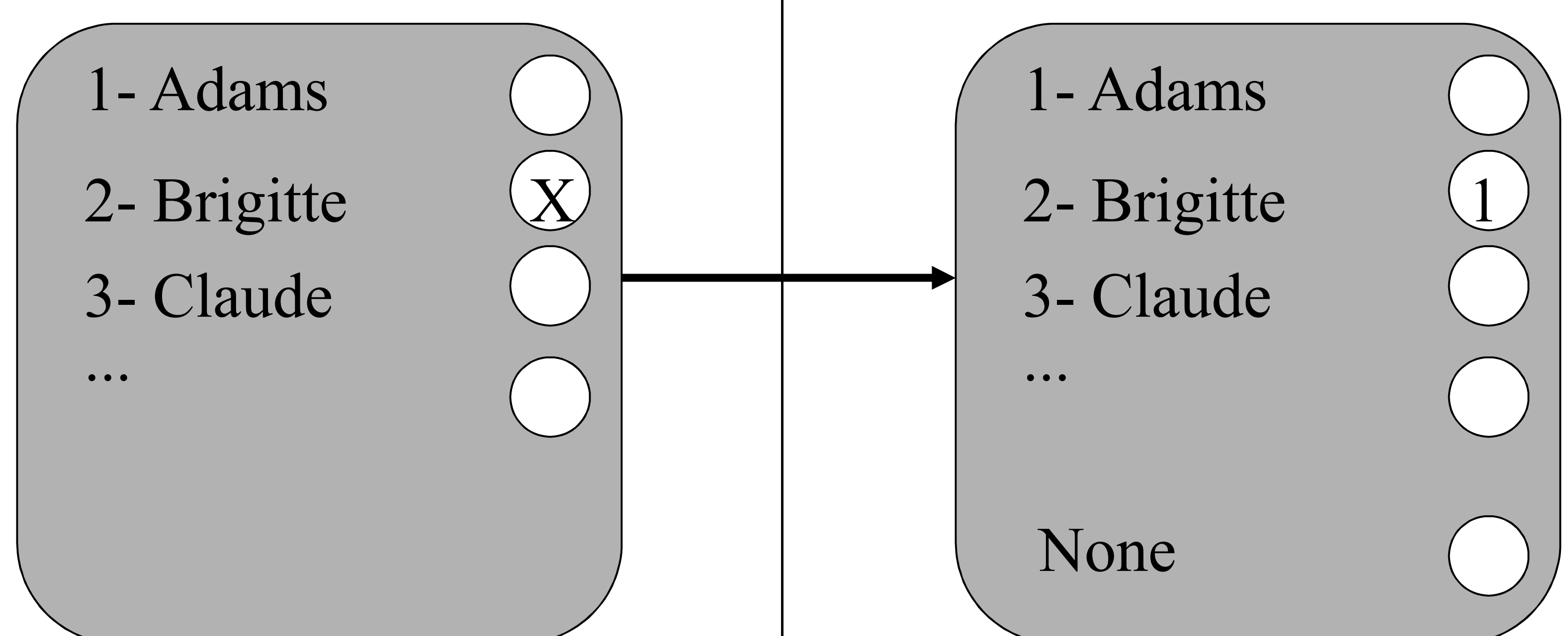
- Express sincere preferences
  - Split acceptable candidates from undesired candidates
  - Order in increasing preferences the acceptable candidates
- Refuse all candidates
- Respect the traditional uninominal ‘‘X’’ approbation



Respect a Traditional Ballot

Ballot marked in a traditional way

Equivalent in preferential terms



Influence of Preferential Ballot over Voting Behavior

- Enhanced government stability
- Unique visit to the polling station
- Individual accountability of the politicians
- Depolarization of the debate
- Higher individual approbation rates

=> Get and express more nuances with simple preferences

Influence of Proportional Representation over Voter Participation

- Same status for independant candidates
- Party line uninstitutionalized
- Less vote splitting issues

⇒ Estimation of 7 to 10 % higher participation rates

- IDEA Institute (Sweden)

A Preferential, Proportional and Acirconscriptive System

(*SPPA in french*)

- For every district:
  - Use elimination rounds like when choosing a party leader
  - Keep final approbation sticking to eliminated candidate
- Compile all final approbations
- For every political party or independant:
  - Average approval support to determine the number of allocated seats
  - Rank candidates to build party list in decreasing support order
  - Elect as many top list candidates as the number of allocated seats

In a District: Allowing to Rally

- Avoid vote-splitting issues using alternative vote (AV)

1st Round

Candidate B	29%	
Candidate A	25%	
Candidate C	22%	
Candidate D	9%	4%
Candidate E	4%	=> Cand. E eliminated
None	11%	=> 11% disaprobation

2nd Round

Candidate B	29%	
Candidate C	26%	
Candidate A	25%	9%
Candidate D	9%	=> Cand. D eliminated
None	11%	=> 11%-11% = 0% support for Candidate E

3rd Round

Candidate C	35%	
Candidate B	29%	6% 13% 6%
Candidate A	25%	=> Cand. A eliminated
None	11%	=> 11%-11% = 0% support for Candidate D

4th Round

Candidate C	48%	3% 32%
Candidate B	35%	=> Cand. B eliminated
None	17%	=> 17%-11% = 6% support for Candidate A

5th Round

Candidate C	51%	=> 51% support for Candidate C
None	49%	=> 49%-17% = 32% support for Candidate B

Results per Districts

- Final approval supports at example District n.2:

Candidate A	6%
Candidate B	32%
Candidate D	51%
Candidate C	0%
Candidate E	0%
None	11%

Party \ District	n.1	n.2	n.3	n.4	n.5	n.6	n.7	n.8	n.9	n.10	Average
Party A	52	6	85	54	6	12	34	39	33	24	34,5
Party B	13	32	6	27	19	12	17	32	31	0	18,9
Party C	9	51	0	3	9	20	19	7	1	22	14,1
Party D	4	0	2	9	11	22	3	10	23	14	9,8
Party E	1	0	0	1	4	4	3	2	4	6	2,5
Independent	0	0	0	0	46	21	10	0	0	23	10
None	21	11	7	6	5	9	14	10	8	11	10,2

Proportional Representation to the Integral Limit

A) Determine first the number of seats for each party (higher rest)

Party A	:	34,5% / 8,98%	=	3,84	vs	4 elected members
Party B	:	18,9% / 8,98%	=	2,1	vs	2 elected members
Party C	:	14,1% / 8,98%	=	1,57	vs	2 elected members
Party D	:	9,8% / 8,98%	=	1,09	vs	1 elected member
Ind S.5	:	4,6% / 8,98%	=	0,51	vs	1 elected member
Party E	:	2,5% / 8,98%	=	0,28	vs	0 elected member
Ind S.10	:	2,3% / 8,98%	=	0,26	vs	0 elected member
Ind S.6	:	2,1% / 8,98%	=	0,23	vs	0 elected member
Ind S.7	:	1% / 8,98%	=	0,11	vs	0 elected member

B) Seats of each party are attributed to its candidates with most support.

Composition of the Parliament:
Party A: Candidates n.3, n.4, n.1 and n.8.
Party B: Candidates n.2 et n.8.
Party C: Candidates n.2 et n.10.
Party D: Candidate n.9.
Independent n.5.

Build the Lists from Results

Classical proportional systems

Pre-established order

1- David
2- Ben
3- Cécile
4- Alicia
...

Resulting list

Order determined from the supports received by each candidates of the same political party

	Votes
Alicia	14 321
Ben	13 467
Cécile	18 234
David	17 534
...	

1- C.
2- D.
3- A.
4- B.
...

Reproducible Tie-Breakers

- Average of final results of equally weighted scenarii with each elimination
  - Fair but heavy process, long to check manually when ties occur in the first rounds
- Simultanuous elimination without internal possibilities of transfer
  - Easier to implement and to check but could harm tied clones
- Rest of euclidian division of the tie support over the number of tied options
  - Easy to check manually
  - Fair to multiple clones
  - Can even break a perfect tie

Tie-Breaker #1: No Rallying Among Ties

100 Ballots

10: A > B > C
10: A > B
10: A
11: B > C > A
9: B > A > C
10: B
9: C > B > A
7: C > A
7: D > E > C
1: D > B
2: D > E
1: E > D > A
8: E > C
1: E
4: none.

1st Round

Candidate A	30	
Candidate B	30	
Candidate C	16	
Candidate D	10	⇒ Cand. D eliminated
Candidate E	10	⇒ Cand. E eliminated
None	4	=> 4 disaprobation

3rd Round

Candidate B	31	
Candidate A	31	
Candidate C	31	
None	7	=> 7 - 4 = 3 support for Candidates D (2) and E (1)

Final Support

Candidate A	31
Candidate B	31
Candidate C	31
Candidate D	2
Candidate E	1
None	4

Tie-Breaker #2: Average of scenarios

1st Round

Candidate A	30
Candidate B	30
Candidate C	16
Candidate D	10
Candidate E	10
None	4

D eliminated

Final Support

Candidate A	18
Candidate B	60
Candidate C	0
Candidate D	0
Candidate E	18
None	4

1/2

E eliminated

3rd Round

Candidate A	31
Candidate B	31
Candidate C	31
None	7

C

Final Support

Candidate A	18
Candidate B	60
Candidate C	15
Candidate D	2
Candidate E	1
None	4

1/6

B

Final Support

Candidate A	21
Candidate B	11
Candidate C	61
Candidate D	2
Candidate E	1
None	4

1/6

A

Final Support

Candidate A	11
Candidate B	60
Candidate C	22
Candidate D	2
Candidate E	1
None	4

1/6

Final Support

Candidate A	17,33
Candidate B	51,84
Candidate C	16,33
Candidate D	1
Candidate E	9,5
None	4

Tie-Breaker #3: Systematic Scenario

1st Round

Candidate A	30
Candidate B	30
Candidate C	16
Candidate D	10
Candidate E	10
None	4

Candidate D ==> 1

Candidate E ==> 0

Euclidian division: 10 = 2 x 5 + **0** thus E is selected as loser.

E eliminated

3rd Round

Candidate A	31
Candidate B	31
Candidate C	31
None	7

Candidate A ==> 1

Candidate B ==> 2

Candidate C ==> 0

Euclidian division: 31 = 3 x 10 + **1** thus A is selected as loser.

A eliminated

Final Support

Candidate A	11
Candidate B	60
Candidate C	22
Candidate D	2
Candidate E	1
None	4

Artificial Intelligence Taking Decisions

- Options as candidates
- Estimators as voters
- Systematic algorithm as electoral process
- Valid for fuzzy environment with multiple same confidence inputs

⇒ Evaluate satisfaction, rank options and chose the best solution

Example : surviving strategy of one or several robots on Mars in case of a sandstorm or exploring Fukushima nuclear plant

- Reproducible tie-breakers

⇒ Valid to debug single-winner AI decision taking algorithm

⇒ Result validation and fraud prevention

Conclusion

- Possible to merge both preferential ballot and proportional representation advantages
- SPPA is one way of doing it
- A reproducible tie-breaker is an essential component for result validation and fraud prevention

In a District: Allowing to Rally

- Other example: district n.4

1st Round

Candidate A	32%	
Candidate B	29%	
Candidate C	17%	
Candidate D	14%	1% 1%
Candidate E	2%	=> Cand. E eliminated
None	6%	=> 6% disaprobation

2nd Round

Candidate A	32%	
Candidate B	30%	
Candidate C	17%	4% 1% 9%
Candidate D	14%	=> Cand. D eliminated
None	7%	=> 7%-6% = 1% support for Candidate E

3rd Round

Candidate B	34%	
Candidate A	33%	9% 5% 3%
Candidate C	17%	=> Cand. C eliminated
None	16%	=> 16%-7% = 9% support for Candidate D

4th Round

Candidate A	42%	12% 27%
Candidate B	39%	=> Cand. B eliminated
None	19%	=> 19%-16% = 3% support for Candidate C

5th Round

Candidate A	54%	=> 54% support for Candidate A
None	46%	=> 46%-19% = 27% support for Candidate B

Non Discriminatory Districts

- Sampling of the electorate – different discretization
  - Last digits of the social insurance number
    - simple for 100 districts
  - Birth dates (day, month, modulo of the year)
    - simple pour 12 seats (municipal)
    - simple for 365 districts
- Advantages
  - No strategical nominations
  - No bribing of the electorate
  - No gerrymandering
  - No regional confrontation
  - Fair representation according to the electorate will

In a District: Allowing to Rally

- Avoid vote-splitting issues using alternative vote (AV)

1st Round

Candidate A	32%	
Candidate B	29%	
Candidate C	17%	
Candidate D	14%	1% 1%
Candidate E	2%	=> Cand. E eliminated
None	6%	=> 6% disaprobation

2nd Round

Candidate A	32%	
Candidate B	30%	
Candidate C	17%	4% 1% 9%
Candidate D	14%	=> Cand. D eliminated
None	7%	=> 7%-6% = 1% support for Candidate E

3rd Round

Candidate B	34%	
Candidate A	33%	9% 5% 3%
Candidate C	17%	=> Cand. C eliminated
None	16%	=> 16%-7% = 9% support for Candidate D

4th Round

Candidate A	42%	12% 27%
Candidate B	39%	=> Cand. B eliminated
None	19%	=> 19%-16% = 3% support for Candidate C

5th Round

Candidate A	54%	=> 54% support for Candidate A
None	46%	=> 46%-19% = 27% support for Candidate B

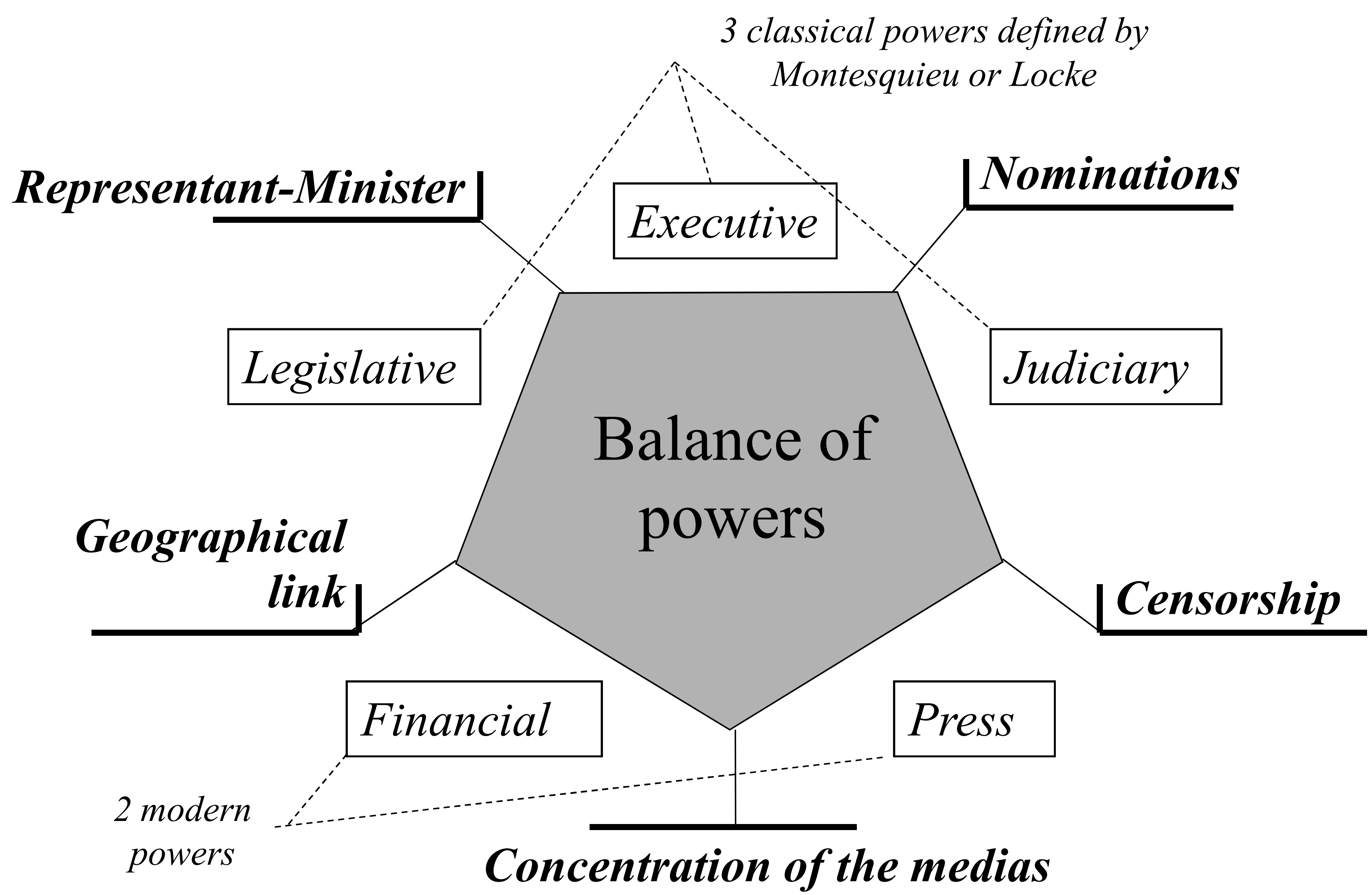
A modular approach

Elements => Qualities

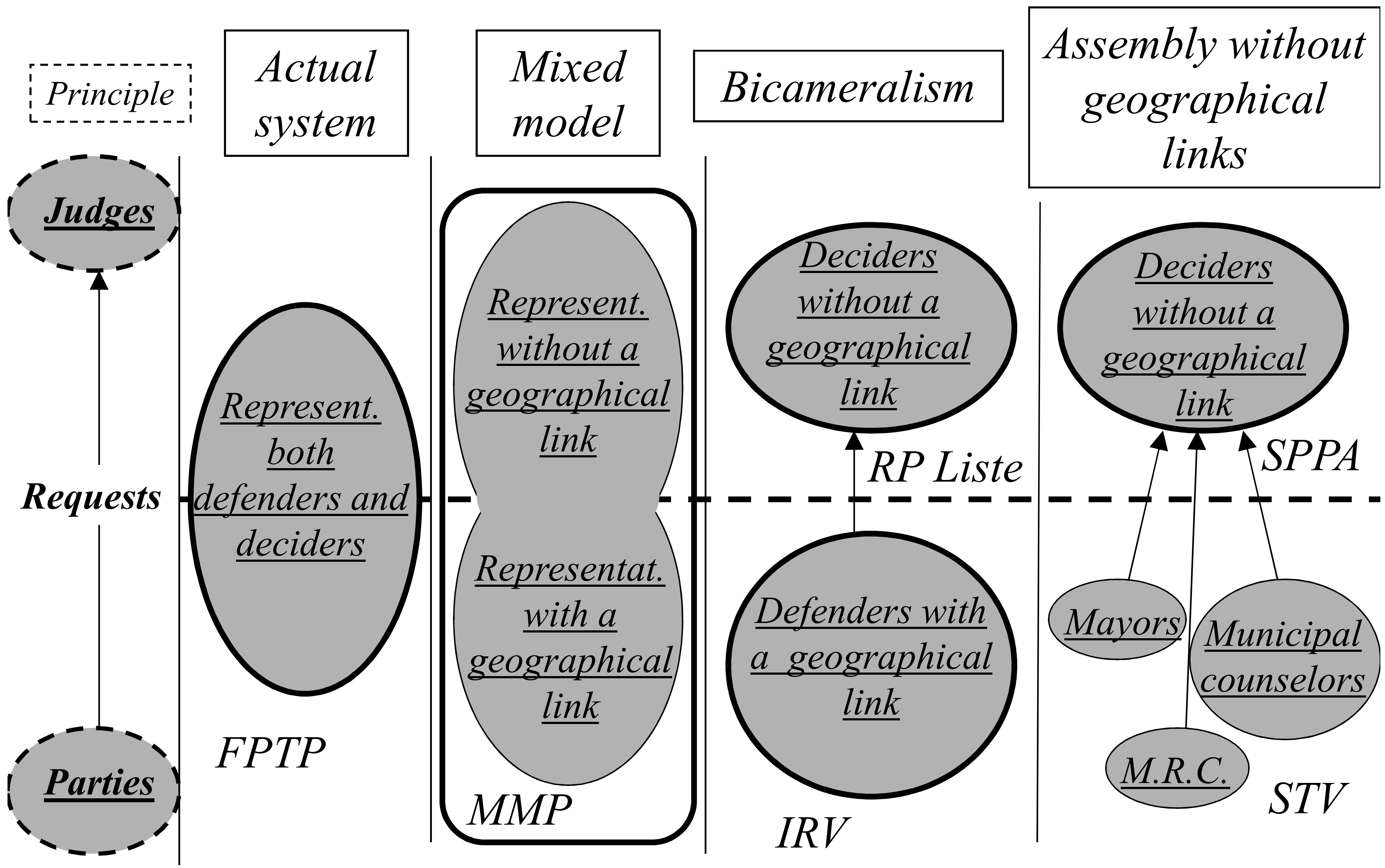
- Preferential ballot
- Mecanism allowing to rally
- Integral proportional representation
- “Crutch” option to garantee at least a coalition of two parties
- Non-discriminatory definition of districts

- still needs only one visit to the polling station
- maintains the accountability link with elected members
- gathers sincere preferences
- allows ordering of the issues by the electorate
- reduce the antagonism between candidates
- vanishes vote-splitting issues
- raises the individual approbation rate of elected members
- treats all candidates equally (independent included)
- eliminates the democratic deficit
- treats all political parties equally (no quota)
- gives to every voter the same weight
- elects party-line builders
- garantees stable coalitions of two parties
- garantees twice the number of MP in the worst case
- preserves learning a small number of candidates to vote
- reduces strategical nominations
- hinders bribing electorate support
- avoids regional confrontation
- eliminates gerrymandering

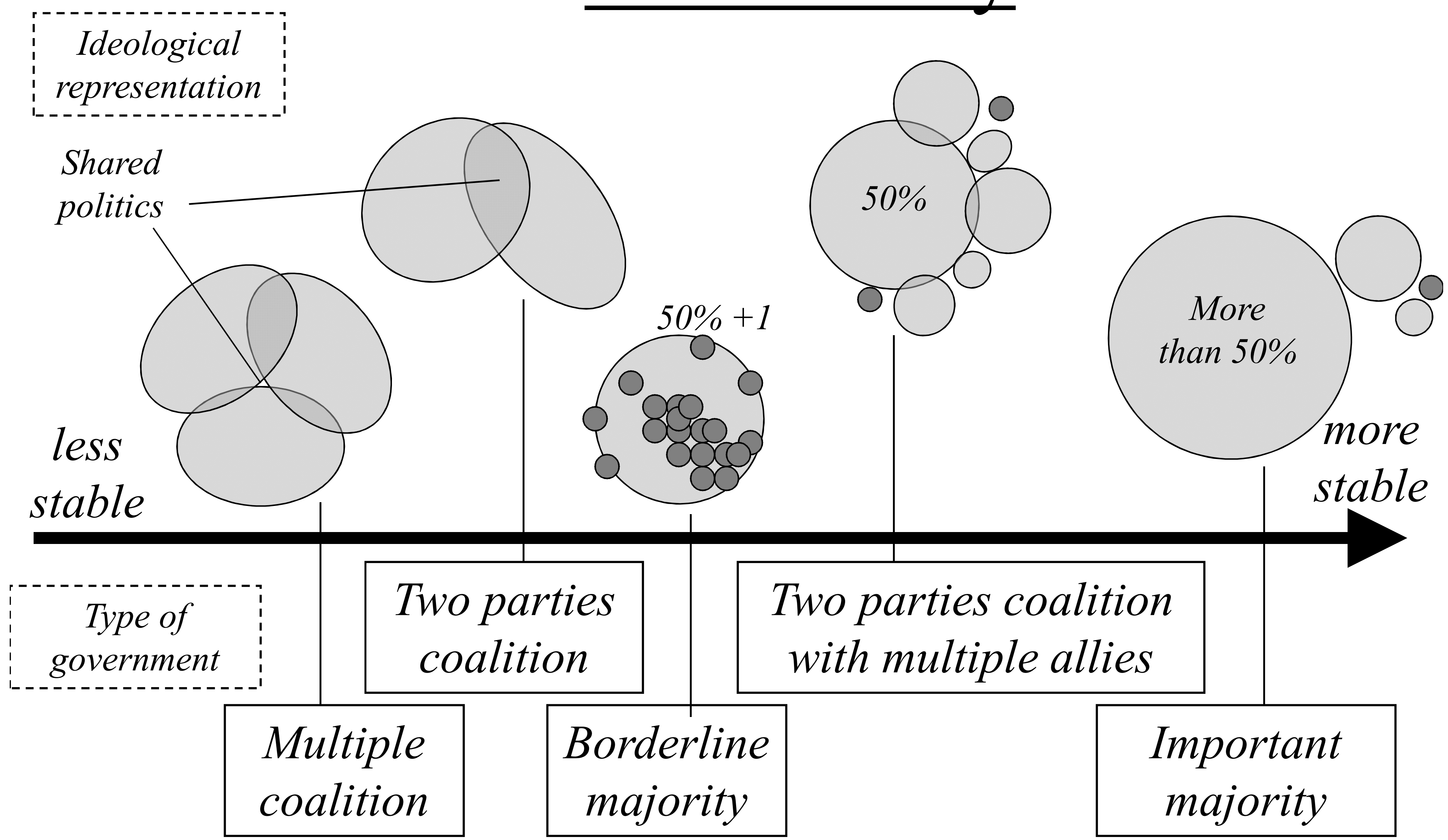
Separate the powers



Separate the roles

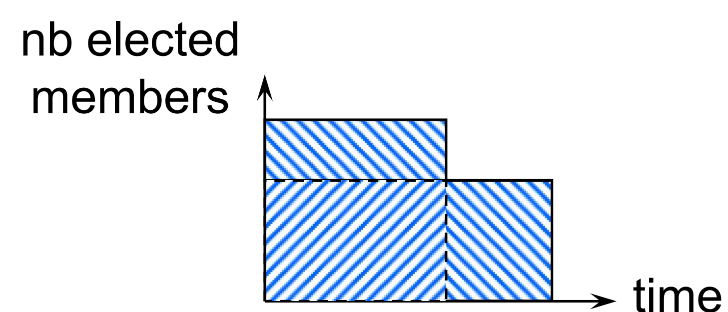


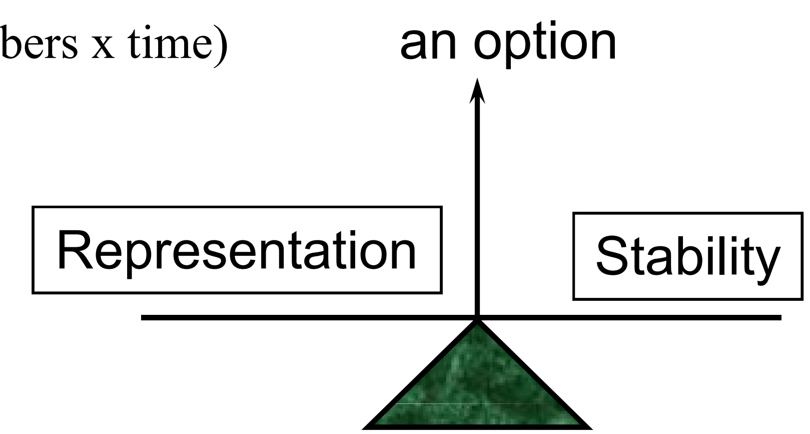
Conciliate representation and stability



Representation vs stability: the “crutch” option

- ❑ Representation exercise
  - – garantee at least coalitions of two parties
  - – compense with additional elected members the plurality party in order to reach $\lfloor 50\% \rfloor$ of the seats
  - – reduce the maximal length of the mandate in proportion:
    - • Preserve the invariant (nb elected members x time)





- ❑ Example:
  - – 30 elected members for an assembly of 70 persons
  - – => add 10 elected members to the winning party
  - – 40 elected members from a total of 80 seats
  - – validity: 30/40 = 75% of the original mandate length

Three steps implementation

- The “crutch” option to garantee stable coalitions of two parties
- Fair representation
  - Preferential ballot
    - applied in Eire (Irish country)
  - Election with rounds allowing to rally and build support
    - applied in Australia
  - Individual proportional representation
    - applied in Finland
- Non-discriminatory definition of districts

FROM PREFERENTIAL INPUT TO PROPORTIONAL OUTPUT

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Keywords: Preferential Ballot, Rallying Method, Leadership Run, Proportional Results, Reproducible Tie-Breakers, Artificial Intelligence.

Brief overview: Preferential ballot usually promotes better behaviours among candidates because it rewards validating the good ideas of an opponent to rally his supporters if this opponent gets eliminated at a later round. However, that concentration can wipe out a segment of the political spectre entirely, like the Green Party for example at the Federal level in Canada. On the other hand, proportional representation induces segmented chambers with many political parties, creating minority governments and potentially unstable coalitions. Including a rallying method used by all political parties in North America during leadership runs, we propose attributing seats proportionally to rallied supports, by building lists from the residual supports of all voters. The resulting method (SPPA) would improve the behavior of voters in several ways. Votes in support of a specific policy could not be diluted by the fact that several candidates are running to defend that policy. An elector could cast a "none" ballot to protest against all the proposed candidates. This result would be interpreted differently from the result produced by absent voters. Allowing voters to rally to the support of a candidate gives both the party in power and the opposition an opportunity to regroup. Despite the large number of candidates, it is quite likely that parliament will only include few political parties. A reproducible tie-breaker is a key element to automatize the counting process for more complex multiple-winner methods like SPPA. Three automated and reproducible tie-breakers are proposed to ensure validation of the results. These tie-breakers can also be useful for making decisions with single-winner methods in an artificial intelligence context.

For concision purposes, the masculine form might be used in this document.

Elections are held daily now everywhere on earth: presidential, legislative or senatorial, at the national, provincial or municipal level, for co-owners, syndicates or student unions... Most of them use simple algorithms and paper to determine a collective choice. These methods are simple, fast and results are easily reproducible. However, democratic pressure is growing to avoid unfair behaviors and consequences resulting from these methods. Vote-splitting, strategic voting and political disinformation are increasingly unacceptable in our modern societies. With the reduced slope of global economy growth, strong debates like the distribution of wealth will increase. The representativeness of the governing assembly will impact the legitimacy of its decision making. In some cases, this may result in the need to improve government democratic representation. Thus, many countries will consider modernizing their electoral systems.

Despite some enhanced electoral methods, a fair and fast counting procedure needs the calculation power of a computer. Sadly, the threat of hacking and fraudulent data tampering are associated with electronic computations. Reproducibility of the counting process by anyone on any computer then becomes a matter of result-legitimacy. Moreover, using computers does not eliminate the possibility of ties during the counting process with preferential ballots. Thus, a safe and reproducible tie-breaking procedure that ensures identical results becomes the key element for introducing these electoral methods in everyday elections.

1. Literature review

In addition to the classical FPTP, the usual methods considered for single-winner elections are approval voting [1], many Condorcet pairwise comparison methods that can use margin like Tideman's typical version [2] or winning votes or relative margin as criteria, range voting described on the internet [3] and median voting [4]. These methods can be generalized for multiple-winner elections as for example proportional approval voting [5] [6] promoted by Forest Simmons, approval residual weights methods (SPPA) [7], Single Tranferable Vote (PR-STV) [8] and Reweighted Range Voting [9]. Several other electoral methods are available on the electorama.com website [10]. Without covering all these methods individually, we will look at different tie-breakers that can be applied and will comment on their application to SPPA. This latest method uses preferential ballots as input and produces proportional results as output.

Even for single-winner elections using the same input ballots, we already know that different electoral systems can provide different results, as described in Malkevitch's example on his webpage [11] and in this book [12]. Details on the different results can be found in the Annals of the New York Academy of Sciences [13]. Thus, it is important to determine which electoral system is used before any election. Most methods need a tie-breaking procedure but some, like FPTP and approval, are simple enough from a counting point of view that the operation can be done by hand. The number of ties is small and only final ties need to be resolved. Thus, it is also important to agree on a tie-breaking procedure before any election or selection to agree on a winning person or decision.

The first step is to identify the different kinds of ties. A regular tie happens when two options obtain the same number of votes as support during a comparison round. If there are more than two options with the same support, it is a multiple-tie. If the tie occurs at the last step of the election, there is no clear winner, and a special treatment is necessary to resolve final ties. In the worst case, a multiple final-tie could even occur. Finally, some ties can be virtual ties when no more comparison rounds will take place. Thus, not all methods listed above need a tie-breaking procedure. To be precise, these methods do not need a tie-breaking procedure for every step of their counting algorithm. Multiple-winner methods that constitute a representation exercise and do not reflect a power shift have this possibility. For example, using SPPA, although a tie-breaking procedure could be needed when comes the time to determine the ranks of each candidate within the party list from the electoral results, no tie-breaker is needed to optimize the respective residual support of candidates who are adversaries in the same district.

Most tie-breakers are not used in electoral cases. It is usually a sport or some other competition issue. While in sport the most common technique consists of overtime, in some cases additional criteria are used. A typical case is the Soccer European Cup with the list of tie-breakers available in section 8.07 [14]:

"If two or more teams are equal on points on completion of the group matches, the following criteria are applied, in the order given; to determine the rankings:

> *a) higher number of points obtained in the matches among the teams in question;*
> *b) superior goal difference in the matches among the teams in question (if more than two teams finish equal on points);*
> *c) higher number of goals scored in the matches among the teams in question (if more than two teams finish equal on points);*
> *d) superior goal difference in all the group matches;*
> *e) higher number of goals scored in all the group matches;*
> *f) position in the UEFA national team coefficient ranking system (see Annex I, paragraph 1.2.2);*
> *g) fair play conduct of the teams (final tournament);*
> *h) drawing of lots."*

A comparison with the 2010 FIFA world-cup tie-breaker rules [15] shows that tie-breakers are different. However, they often refer to additional outside information and finally end up with a random toss-up. If this extent can be acceptable to determine a qualification, it is not desirable to identify a semi-final or final winner. Since it last occurred in Rome in 1960, when the Yugoslavian team won the semi-final by drawing lots, final toss-up has been avoided. Tie-breaker criteria g) and h) above could not be reached from the Euro previous list because f) was a definitive criterium (all UEFA national team coefficient rankings were different).

An example of a tie-breaker for electoral systems is provided by the selection of the US president. Having this important collective decision ending in a toss-up would not be acceptable to most. Accessing another representative assembly to further discriminate a winner is another solution to break a final tie [16]:

"US Presidential Election Tie-Breaker

If no candidate receives a majority of the Electoral College votes in a US Presidential election, the states' delegations to the House of Representatives select the president. Each state's delegation receives one vote. The House must select from the top three Electoral College vote getters (i.e. the three candidates with the highest Electoral College vote totals), and the winner must receive the majority of votes.

A minimum 2/3rds quorum (i.e. 2/3rds of the states's delegations must be present, and the *winner must get a simple majority of that quorum). Only state delegations can vote in such a tie-breaker (e.g. the District of Columbia's Electoral representatives are excluded, and D.C. does not get a vote). Voting rounds continue until there is a winner."*

Some alternatives exist, for example electing winners with a tie-breaker (from a different assembly or a random toss-up) for a temporary term, until a new election is conducted. The most important aspect is that the procedure be described within the constitution before the election starts. The consequences of fixed date elections should be taken into account for example. In summary, an automatized counting procedure should not use random toss-up nor refer to additional outside information. The only exception could be for final ties, but it is not recommended.

2. Multiple tie-breaking and multiple ties

While the most unpopular defects of the current electoral systems can be reduced with more precise ballots and more complex algorithms, the relative simplicity of ballot filling can be preserved. However, even if ranking or grade ballots can be filled relatively easily, the counting procedure becomes longer and the number of ties grows. Not only can several ties occur more often during an elimination process (with preferential, STV or IRV cases for example), but multiple ties involving more than two candidates need to be treated as well. Nevertheless, speed and reproducibility of the counting steps that lead to the determination of winners can be maintained with computers if we define an appropriate tie-breaker.

For algorithmic use, the ideal tie-breaking procedure should verify the following four criteria:

- Generic (ability to handle multiple candidate ties);
- Fair (same probability for every candidate);
- Reproducible (always the same result);
- Safe (no external information).

Although extremely rare, multiple-ties need to be resolved with the same thoroughness as regular ties to produce an algorithm process that will not crash while counting ballots (or identifying a winner by another mathematical process, with a median for example). Thus, the tie-breaker should be designed to isolate a candidate according to the round when the tie occurs. Typically, for an FPTP election the tie-breaker should identify a winner, but for an IRV round, the tie-breaker should be used to identify a loser. Again, a clear agreement on the tie-breaker should be defined prior to the election and the case of a multiple-tie is the simpler way to identify what should be the final and direct objective of the tie-breaking procedure. From cases covered by the literature review, we see that the resolution of not only multiple-ties but final ties can be useful. Obviously, in order to be accepted by all candidates, fairness is fundamental and all candidates involved in a tie shall have the same probability of being discriminated (as a winner or a loser according to the electoral method).

The election database includes all ballots. Database tempering is a different matter and appropriate protection is mandatory. The electorama.com [18] community tends to favor a mixed approach: the combination of an electronic counting procedure to accelerate the process and a paper archive for later validation. Thus, hackers should address both systems in order to modify the database and the result of an election. To make sure no fraud prevails at the tie-breaking procedure, randomness is not acceptable as it cannot be neither verified on demand nor reproduced by anyone. A safe tie-breaker will use no external information to obtain a deterministic tie-break. Finally, the safe, reproducible, fair and generic tie-breakers need to fit the method proposed. Three tie-breakers are proposed to complement SPPA algorithm.

<u>Tie-breaker #1: Simultaneous Treatment of Tied Options</u>
Can sort multiple ties
Rather simple calculations but many exceptions within the implementation
Relatively simple by definition, reproducible
Cannot solve a final tie
Compatible with time sharing of a mandate in case of a final tie.

<u>Tie-breaker #2: Weighted Results of Relevant Scenarios</u>
Can sort multiple ties
Heavy calculations, management of a tree of scenario
Relatively simple by definition, reproducible
Cannot solve a final tie
Compatible with time sharing of a mandate in case of a final tie.

<u>Tie-breaker #3: Euclidian Remainder from Lexicographic Ordering of Options</u>
Can be adapted to directly determine winner or loser from multiple ties
Simple calculations
Simple, fast, reproducible
Can solve a final tie

3. Preferential ballots to produce proportional results

Let see how these tie-breakers behave with a method that uses preferential ballots to produce a proportional result, typically SPPA. First, let's summarize the algorithm when essentially no tie occurs with another example compatible with the scenario described in this reference [7].

3.1- The preferential vote or ordinal ballot is used

The preferential ballot allows an elector to vote for several election rounds, in only one visit to the polling station. For example:

District No.2	
Candidate A	
Candidate B	3
Candidate C	2
Candidate D	1
Candidate E	
None	

In the example above, the elector contributes to district No. 2 representation by first separating acceptable candidates (B, C and D) from undesirable candidates (A and E). Next, he ranks the acceptable candidates according to his preferences: our elector votes for candidate D and indicates that he would be willing to rally to candidate C if D is not available anymore, and later to candidate B if neither D nor C are available. The "None" box allows electors who feel all candidates are undesirable to clearly express their opinion and has different consequences than a "Blank" vote. The elector's action is simple and easy to interpret. A compact representation for this ballot is D > C > B.

3.2- The vote follows the leadership run-off system with rallying

This electoral system is also called Alternative Vote (AV) or Instant Runoff Vote (IRV). At each "Round", the elector's vote is attributed to the first candidate still running from his preference list: the candidate with the least votes is then eliminated. At the next round, his votes are redistributed, until only one candidate remains. Each elector supports only one candidate at the end. Evaluating residual supports for the final result, his vote is attributed to the last candidate he agreed to rally to. Example of district No.2:

1st Round		
Candidate B	29%	
Candidate A	25%	
Candidate C	22%	
Candidate D	9%	
Candidate E	4%	====> Candidate E is eliminated.
None	11%	====> 11% of "None" votes as final result.

2nd Round		
Candidate B	29%	
Candidate C	26%	
Candidate A	25%	
Candidate D	9%	====> Candidate D is eliminated.
None	11%	====> 11%-11% = 0% for Candidate E as final result.

3rd Round

Candidate C	35%	
Candidate B	29%	
Candidate A	25%	====> Candidate A is eliminated.
None	11%	====> 11%-11% = 0% for Candidate D as final result.

4th Round

Candidate C	48%	
Candidate B	35%	====> Candidate B is eliminated.
None	17%	====> 17%-11% = 6% for Candidate A as final result.

5th Round

Candidate C	51%	====> 51% for Candidate C as final result.
None	49%	====> 49%-17% = 32% for Candidate B as final result.

Final Supports:

Candidate A	6%
Candidate B	32%
Candidate C	51%
Candidate D	0%
Candidate E	0%
None	11%

The elector described in point 3.1- votes for candidate D in the two first rounds. In the third round, since candidate D (his first choice) is eliminated, our elector becomes one of the 9% (35%-26%) of voters who rally to candidate C (his second choice). Our elector will approve this choice until last round. As a result for final supports, he votes for candidate C, just like 51% of the electorate.

3.3- The proportional representation is optimal

Let us examine an example of final supports for all districts (assuming there are 10 districts in this case). The last column indicates for each party the average of the votes over all districts.

Party\ Riding	n.1	n.2	n.3	n.4	n.5	n.6	n.7	n.8	n.9	n.10	Average
Party A	52	6	85	54	6	12	34	39	33	24	34.5
Party B	13	32	6	27	19	12	17	32	31	0	18.9
Party C	9	51	0	3	9	20	19	7	1	22	14.1
Party D	4	0	2	9	11	22	3	10	23	14	9.8
Party E	1	0	0	1	4	4	3	2	4	6	2.5
Independent	0	0	0	0	46	21	10	0	0	23	10
None	21	11	7	6	5	9	14	10	8	11	10.2

Table 3.1

A) We start by evaluating the number of seats for each party

We want the 10 seats to be distributed proportionally. Using greatest remainder criteria, the number of seats per political party is:

Party A	:	34.5%	/ 8.98%	=	3.84	vs	4 elected officials
Party B	:	18.9%	/ 8.98%	=	2.1	vs	2 elected officials
Party C	:	14.1%	/ 8.98%	=	1.57	vs	2 elected officials
Party D	:	9.8%	/ 8.98%	=	1.09	vs	1 elected official
Ind S.5	:	4.6%	/ 8.98%	=	0.51	vs	1 elected official
Party E	:	2.5%	/ 8.98%	=	0.28	vs	0 elected official
Ind S.10	:	2.3%	/ 8.98%	=	0.26	vs	0 elected official
Ind S.6	:	2.1%	/ 8.98%	=	0.23	vs	0 elected official
Ind S.7	:	1%	/ 8.98%	=	0.11	vs	0 elected official

Singleness of the representation: to find the minimal error, round down the number of seats to the nearest integer. However, some seats may be empty. Hence, we assign additional seats one by one in order to reach the expected total number of seats (10 in our example). We increase by one the number of elected officials from the party with the highest fractional part and we repeat using decreasing fractional parts. Of course, independent candidates are considered alone... In the event of equal fractional parts, the party with the highest representation gets an advantage (3.46 seats vs 2.46 seats become respectively 4 seats vs 2 seats). If the equality is exact, the leader of the party with the most votes, other than the ones concerned, could pick a winner(s). To ensure reproducibility, this latest step can be replaced using tie-breaker #3 applied on the number of ballots each tied **party** received.

B) Seats of a party are assigned to candidates with the best final support

Thus, several officials can be elected for a district or none at all. Presented in decreasing order of votes, from Table 3.1:

Composition of the Parliament:
Elected officials from party A are its candidates in districts n.3 (85%), n.4 (54%), n.1 (52%) and n.8 (39%).
Elected officials from party B are its candidates in districts n.2 (32%) and n.8 (32%).
Elected officials from party C are its candidates in districts n.2 (51%) and n.10 (22%).
The elected official from party D is its candidate in district n.9 (23%).
The independent in district n.5 (46%) is elected.

Districts n.2 and n.8 produce two elected officials each and districts n.6 and n.7 none. In general, results should be distributed in a more regular way, but the example shows how the model can solve the worst distortions. It is less visible with this very irregular example, but the districts with no representative are often those where the electorate refused to rally or voted for none. In the event of equality between candidates of a same party for the final attribution of seats, the leader of the party could pick the winner(s). Again, to ensure reproducibility, this latest step can be replaced by using tie-breaker #3 applied on the number of ballots each tied **candidate** received.

Further comments regarding either the number of elected representative per district or the stability of a parliament elected using SPPA can be found in reference [7].

4. Application of tie-breakers to resolve ranked elimination process

Some final tie cases can occur during steps 3.3 above, but we can circumvent them either by using the judgement of the best representative exterior to the tie or tie-breaker #3. For steps 3.2, the time restrictions and the number of possible ties suggest a more automated way of resolving ties. Let's use an example to illustrate the results for another district X using the three different tie-breakers proposed in section 2. First, using compact representation presented in subsection 3.1, here are the 100 enhanced preferential ballots:

10 : A > B > C
10 : A > B
10 : A
11 : B > C > A
9 : B > A > C
10 : B
9 : C > B > A
7 : C > A
7 : D > E > C
1 : D > B
2 : D > E
1 : E > D > A
8 : E > C
1 : E
4 : none.

1st Round	
Candidate A	30
Candidate B	30
Candidate C	16
Candidate D	10
Candidate E	10
None	4 ====> 4 for “None” votes as final result.

SPPA uses elimination rounds, thus any tie-breaker should be used to identify a loser option. Two regular ties appear in the first round results, but we will apply tie-breakers only to the D-E tie, hoping that the other virtual tie will simply vanish.

4.1- Tie-breaker #1: Simultaneous Treatment of Tied Options

Both losers (D and E) are eliminated simultaneously without possibility of rallying between both candidates. The process produces directly the third round results.

<u>3rd Round</u>

Candidate A 31
Candidate B 31
Candidate C 31
None 7 ====> 1 vote for Candidate E (1 : E)
and 2 votes for Candidate D (2 : D > E) as final result.

A triple equality appears. Because no candidate remains for rallying, that triple equality is a final tie. Because SPPA uses these residual supports to implement proportional results, this tie is virtual and it is not mandatory to resolve it. The final residual approbation supports in votes are thus for district X:

<u>Final Supports with Tie-breaker #1</u>:

Candidate A 31
Candidate B 31
Candidate C 31
Candidate D 2
Candidate E 1
None 4

4.2- Tie-breaker #2: Weighted Results of Relevant Scenarios

In the case of a tie for last place in any round, each scenario is played out and the weighted average of the results is retained as a result of final supports. All losers will generate an equally weighted scenario with the possibility of rallying to any other losing option. The process produces second round results for scenario 1 (E eliminated) and 2 (D eliminated).

<u>2nd Round of Scenario 1</u>

Candidate A 30
Candidate B 30
Candidate C 24
Candidate D 11
None 5 ====> 1 vote for Candidate E as final result of scenario 1.

3rd Round of Scenario 1

Candidate A 31
Candidate B 31
Candidate C 31
None 7 ====> 2 votes for Candidate D as final result of scenario 1.

Again, the triple equality appears. Tie-breaker #2 can resolve the situation, thus three scenarios are created as extensions of scenario 1: scenario 11 (C eliminated), scenario 12 (B eliminated) and scenario 13 (A eliminated). The final residual approbation supports in votes for scenario 1 will be the average of a third of the final supports obtained for each extension. Thus, for district X:

4th Round of Scenario 11

Candidate B 40
Candidate A 38
None 22 ====> 15 votes for Candidate C as final result of scenario 11.

5th Round of Scenario 11

Candidate B 60
None 40 ====> 18 votes for Candidate A as final result of scenario 11.

Thus, final supports of scenario 11 for district X are:

Final Supports with Tie-breaker #2 for Scenario 11:

Candidate A	18
Candidate B	60
Candidate C	15
Candidate D	2
Candidate E	1
None	4

In the same manner, scenario 12 for district X:

4th Round of Scenario 12

Candidate C 42
Candidate A 40
None 18 ====> 11 votes for Candidate B as final result of scenario 12.

5th Round of Scenario 12

Candidate C 61
None 39 ====> 21 votes for Candidate A as final result of scenario 12.

Thus, final supports of scenario 12 for district X are:

Final Supports with Tie-breaker #2 for Scenario 12:

Candidate A	21
Candidate B	11
Candidate C	61
Candidate D	2
Candidate E	1
None	4

Finally, for scenario 13 of district X:

4th Round of Scenario 13

Candidate B 51
Candidate C 31
None 18 ====> 11 votes for Candidate A as final result of scenario 13.

5th Round of Scenario 13

Candidate B 60
None 40 ====> 22 votes for Candidate C as final result of scenario 13.

Thus, final supports of scenario 13 for district X are:

Final Supports with Tie-breaker #2 for Scenario 13:

Candidate A	11
Candidate B	60
Candidate C	22
Candidate D	2
Candidate E	1
None	4

Averaging all final results of extended scenarios to determine the supports for scenario 1:

Final Supports with Tie-breaker #2 for Scenario 1:

Candidate A	16,67
Candidate B	43,67
Candidate C	32,67
Candidate D	2
Candidate E	1
None	4

Completing the procedure for scenario 2 is easier because no more ties appear, thus it has no sub-scenarios:

<u>2nd Round of Scenario 2</u>

Candidate B	31	
Candidate A	30	
Candidate E	19	
Candidate C	16	
None	4	====> No vote for Candidate D as final result of scenario 2.

<u>3rd Round of Scenario 2</u>

Candidate B	40	
Candidate A	37	
Candidate E	19	
None	4	====> No votes for Candidate C as final result of scenario 2.

<u>4th Round of Scenario 2</u>

Candidate B	40	
Candidate A	38	
None	22	====> 18 votes for Candidate E as final result of scenario 2.

<u>5th Round of Scenario 2</u>

Candidate B	60	
None	40	====> 18 votes for Candidate A as final result of scenario 2.

Summarizing final supports for scenario 2 of district X produces:

<u>Final Supports with Tie-breaker #2 for Scenario 2</u>:

Candidate A	18
Candidate B	60
Candidate C	0
Candidate D	0
Candidate E	18
None	4

Averaging scenario 1 and 2 or using the weights of every leaf-scenario of the extended tree (1/2 for scenario 1 and 1/6 for scenarios 11, 12 and 13), results are identical using tie-breaker#2 for district X:

<u>Final Supports with Tie-breaker #2</u>:

Candidate A	17,33
Candidate B	51,84
Candidate C	16,33
Candidate D	1
Candidate E	9,5
None	4

If there was a tie at this point, it would be considered a virtual tie because SPPA applies proportional representation at next step.

4.3- Tie-breaker #3: Euclidian Remainder from Lexicographic Ordering of Options

Back to the first round that generates a tie between candidates D and E:

1st Round

Candidate A	30	
Candidate B	30	
Candidate C	16	
Candidate D	10	
Candidate E	10	
None	4	====> 4 for "None" votes as final result.

Starting at the same first round, we associate a remainder in lexicographical order to every candidate involved in the tie (final remainder is always 0):

Candidate D ==> 1
Candidate E ==> 0
Euclidian division: 10 = 2 x 5 + **0** thus E is selected as loser.

2nd Round

Candidate A	30	
Candidate B	30	
Candidate C	24	
Candidate D	11	
None	5	====> 1 vote for Candidate E as final result.

3rd Round

Candidate A	31	
Candidate B	31	
Candidate C	31	
None	7	====> 2 votes for Candidate D as final result.

Again, we associate a remainder in lexicographical order to every candidate involved in the tie (final remainder is always 0). Euclidian divisor is 3 because there is a triple tie:

Candidate A ==> 1
Candidate B ==> 2
Candidate C ==> 0
Euclidian division: 31 = 3 x 10 + **1** thus A is selected as loser.

4th Round

Candidate B	51	
Candidate C	31	
None	18	====> 11 votes for Candidate A as final result.

5th Round

Candidate B	60	
None	40	====> 22 votes for Candidate C as final result.

Final Supports with Tie-breaker #3:

Candidate A	11
Candidate B	60
Candidate C	22
Candidate D	2
Candidate E	1
None	4

The summary of final supports using tie-breaker #3 for district X shows that it arbitrarily but systematically generates one of the previous scenario of tie-breaker #2 (scenario 13 in this case). Tie-breaker #1 tries to minimize rallying. Tie-breaker #2 tries to gather all information, it can even detect some residual support for candidate E in scenario 2. In the context of SPPA, every tie-breaker generates very different results. This example illustrates why the tie-breaking procedure should be determined before the election starts.

5. A digression about artificial intelligence (AI)

Recent developments in artificial intelligence create a context with similar problematics. A typical problem involving intelligence artificial comes from emergency cases for robots sent by NASA on other planets. The nearest one, Mars, involves a communication delay of 8 minutes between the planet and Earth control center. In case of a marsquake (an earthquake on Mars) or a sandstorm, we need some faster process to take decisions. An autonomous system to gather information, compare data and take actions seems the best way to preserve precious robots. Even if the end-goal is not the same, in such cases, an artificial intelligence would take decisions using a single-winner voting system.

A typical automated system does not vote. It relies on an optimization algorithm that starts from well known facts and applies rigorous logic to get to the next step. The anticipated succession of these steps performs the expected task, with the greatest expected efficiency. For example, based on the positions and movements of the other elevators, the determination of the floor where an elevator should wait to reduce the waiting time of their users is an optimization problem.

For non-critical decisions, we usually accept a suboptimal algorithm. But how do we expect very expensive robots to behave in a fuzzy environment? The main difference between automated and autonomous systems comes from the difficulty humans would have to interact on the system to repair or preserve it. It is the case for multi-million dollars probes and robots used to scout the universe and for very useful robots we use to probe dangerous environment like the ones we sent to explore Fukushima nuclear plant. In such cases, a fast and automated process to take decisions helps to determine which behavior adopt to return the robot in a safe place, to repair it and extend its lifetime. In a well-known environment, a reliable measurement can trigger a simple alarm. But in a fuzzy environment, how to interpret multiple inputs coming from damaged sensors (because of radiations or a sandstorm)? Rigorous optimization algorithms do not longer apply and we can move to voting systems to provide a stochastic solution.

In a single winner decision context, the example of multiple sensors on an inaccessible robot can be used: in the case of a sandstorm on Mars, visible cameras, infra-red cameras, microphone, anemometer, vibration detector, barometer, thermometer could all provide a different ranking of strategies to fulfill the task:

- Keep working;
- Go to preventive shutdown;
- Hide in some safer place.

In some future, multiple cobots [18] (cooperative robots), with different inputs based on location and health, could even suggest more strategies and decide to:

- Regroup;
- Repair each other;
- Build some protection.

To obtain these answers and optimize the life-expectancy of these expensive autonomous systems, we need to simulate and to reproduce artificial decisions. Reproducible tie-breakers then become a key element for debugging voting systems in an artificial intelligence context. The same techniques artificial intelligence could use to take decisions in a fuzzy environment could be used in the context of humans for multiple-winner elections: robotic considerations for selecting a decision meet the democratic preoccupations of mankind for electing a person.

6. Conclusion: impacts on the behaviors of voters

SPPA electoral system uses preferential ballots to produce proportional results. The results should better represent the will of the electors by reducing the random effects and the biased strategies. In addition, this system is applicable to a complementary election. Seats already taken since general election are used as basis and the empty ones can be filled using complementary procedure of MMP to correct for proportionality according to supports for different political parties at the date of the complementary election.

Votes in support of a specific policy cannot be diluted by the fact that several candidates are running to defend that policy. Allowing voters to rally to the support of a candidate gives an opportunity to regroup both the party in power and the opposition. Despite the large number of candidates, the parliament should only include few political parties.

In the context of SPPA, every tie-breaker generates very different results. Tie-breaker#1 tries to minimize rallying and freezes final ties. Tie-breaker#2 tries to gather all information, it is the most precise but there is no guarantee the number of scenarios won't explode. Tie-breaker#3 randomly but systematically selects one scenario: it is not perfect from a fairness point of view but it is simple, efficient and reliable.

These tie-breakers are reproducible. Any data scientist could reproduce and validate the results of any election using a published database of the ballots. Recording a pseudonym linked to his ballot, any voter could verify his ballot was taken into account. A voter could even track the support provided by his ballot and estimate the size of shifting ballots to modify the result. This precise feedback information could help him understand the impact of its vote and adapt his decision. With any of these tie-breakers, SPPA becomes a reliable electoral system that produces proportional output from preferential input.

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